

Spin relaxation and antisymmetric exchange in *n*-doped III-V semiconductors

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Recently K. Kavokin [Phys. Rev. **B 64**, 075305 (2001)] suggested that the Dzyaloshinskii-Moriya interaction between localized electrons governs slow spin relaxation in *n*-doped GaAs in the regime close to the metal-insulator transition. We derive the correct spin Hamiltonian and apply it to the determination of spin dephasing time using the method of moments expansion. We argue that the proposed mechanism is insufficient to explain the observed values of the spin relaxation time.

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Observation of exceedingly long spin relaxation times for electrons optically pumped into the conduction band of *n*-doped GaAs [1] prompted suggestions for the use of polarized spins in quantum computing [2]. No consensus has been reached, however, on the mechanisms governing the spin relaxation processes. Low-temperature spin dephasing time τ_s in Ref. [1] behaves non-monotonically as a function of donor concentration n_D , reaching values above 100 ns at $n_D = 10^{16} \text{ cm}^{-3}$. This concentration still belongs to the nonmetallic regime.

In an interesting paper [3] the author introduced a new dephasing mechanism acting between the spins of electrons localized on randomly distributed shallow donors. According to Ref. [3], dephasing would occur each time the spins of the two electrons on neighboring centers rotate against each other in an effective spin-orbital field in the process of tunneling between the two centers during the spin flip-flop act caused by the exchange interaction. This notion and the way it was treated in Ref. [3] in terms of the exchange narrowing does not seem to be fully self-consistent. The point is that “random effective fields” originating in [3] from the antisymmetric Dzyaloshinskii-Moriya (DM) [4, 5] interaction and the dynamical averaging due to the isotropic exchange in fact both arise from the interaction of the spins of the very same pair and hence are not independent.

As a more consistent approach to the problem we tried its formulation in terms of the EPR line shape for the spin Hamiltonian which for semiconductors without an inversion center is a sum of the conventional isotropic and the antisymmetric DM exchange terms. The latter is responsible for non-conservation of the total spin of the system.

The antisymmetric interaction

$$\hat{\mathcal{H}}_{\text{DM}} = \mathbf{d}(\mathbf{R})(\hat{\mathbf{s}}_1 \times \hat{\mathbf{s}}_2) \quad (1)$$

in semiconductors lacking the inversion symmetry (III-V and II-VI compounds) is of interest in itself, but has not been properly calculated yet. In Ref. [3] the asymptotic behavior of $\mathbf{d}(\mathbf{R})$ at large distances between centers $R \gg a_B$, where a_B is the effective Bohr radius, for shallow (hydrogen-like) donor centers was calculated in the Heitler-London approximation. But it has been shown

[9, 10] that the Heitler-London theory does not account for the Coulomb correlations inside the “tunneling corridor” connecting the two centers which dramatically affects the prefactor of the isotropic exchange.

Below we derive the asymptotic form of $\mathbf{d}(\mathbf{R})$ at $R \gg a_B$. We have slightly modified the approach of Ref. [9, 10] to include the initial spin degeneracy of the problem and its lifting by interactions in the presence of a weak spin-orbital interaction

$$\hat{\mathcal{H}}_{\text{so}} = \mathbf{h}(\hat{\mathbf{p}})\hat{\mathbf{s}}. \quad (2)$$

Our theoretical result for the R -dependence of the antisymmetric exchange $\mathbf{d}(\mathbf{R})$ differs significantly from Ref. [3, 7, 8]. In Ref. [7] the authors concluded that their experimental results for the spin relaxation time τ_s together with the data [1, 6] agree well with the mechanism [3] in the concentration range $n_D = 2 \times 10^{15} \div 2 \times 10^{16} \text{ cm}^{-3}$. Upon substituting into the same formulae of [3, 7], our corrected DM term increases the estimate for τ_s by a factor of 4. Although this does not rule out completely the relaxation mechanism [3] because of the phenomenological character of the analysis of the experimental data in [7], the possibility that the antisymmetric exchange alone accounts for the values of τ_s observed in [1, 6, 7] in this doping interval seems rather unlikely.

The essence of the method is as follows. The envelope of the wavefunction of an electron localized on a donor impurity in the effective mass theory satisfies Schrödinger equation with the Hamiltonian

$$\hat{\mathcal{H}} = \mathbf{p}^2/2m_e + \hat{\mathcal{H}}_{\text{so}} + e^2/\kappa r, \quad (3)$$

where m_e is the effective electron mass, κ is the permittivity. (For GaAs $m_e \approx 0.072m_0$ and $\kappa \approx 12.5$).

The spin-orbital Hamiltonian (2) in zinc-blende semiconductors has the form [11, 12]

$$h_x(\mathbf{p}) = \frac{\alpha_{\text{so}}}{m_e \sqrt{2m_e E_g}} p_x (p_y^2 - p_z^2). \quad (4)$$

$h_y(\mathbf{p})$ and $h_z(\mathbf{p})$ are obtained by permutation in (4). Here E_g is the band gap and α_{so} is a phenomenological parameter. For GaAs $E_g \approx 1.43 \text{ eV}$ and $\alpha_{\text{so}} \approx 0.07$.

Without the spin-orbital interaction the ground state had the energy $\frac{1}{2}E_0 = -Ry = -m_e e^4 / 2\kappa^2$. ($E_0 \approx -12.6$ meV for GaAs). The corresponding eigenfunction of (3) were $\varphi_\mu^\alpha(\mathbf{r}) = \varphi_0(r)\delta_\mu^\alpha$, where $\delta_\mu^\alpha \equiv \delta_{\uparrow,\downarrow}^\alpha$ denotes the spinors for up and down spin projection,

$$\varphi_0(r) = \frac{1}{\sqrt{\pi a_B^3}} e^{-r/a_B}, \quad (5)$$

and $a_B = \kappa/m_e e^2$ is the Bohr radius ($a_B \approx 92$ for GaAs). In the presence of a spin-orbital interaction the spin projection no longer commutes with (3), but the two-fold (Kramers) degeneracy of the ground state remains. For small spin-orbital interaction (4) eigenfunctions of (3) are

$$\varphi_\mu^\alpha(\mathbf{r}) \approx \varphi_0(r)(e^{ime\hbar\mathbf{h}(\hat{\mathbf{r}})r\sigma/2a_B^2})_{\alpha\mu}, \quad (6)$$

where $\hat{\mathbf{r}} = \mathbf{r}/r$. The phase factor turns into unity when $r \rightarrow 0$. Thus the Kramers index μ takes on the meaning of a spin projection of an electron on the center.

The two-electron Hamiltonian is obtained by combining two single-electron Hamiltonians (3) and the Coulomb interaction of electrons between themselves and with the alien ions:

$$\begin{aligned} \hat{\mathcal{H}}_{\text{tot}} = & (\mathbf{p}_1^2 + \mathbf{p}_2^2)/2m_e + \hat{\mathcal{H}}_{\text{so1}} + \hat{\mathcal{H}}_{\text{so2}} \\ & + \frac{e^2}{\kappa} \left(\frac{1}{r_{12}} + \frac{1}{R} - \frac{1}{r_{1A}} - \frac{1}{r_{1B}} - \frac{1}{r_{2A}} - \frac{1}{r_{2B}} \right). \end{aligned} \quad (7)$$

Here $r_{1A,B}$ and $r_{2A,B}$ denote the distances between the electrons and the two donor centers A and B.

For isolated donors, i.e. on large distances $R \gg a_B$ between two centers a two-electron wavefunction $\psi_{\mu\nu}^{\alpha\beta}$ is merely a product $\varphi_\mu^\alpha(\mathbf{r}_{1A})\varphi_\nu^\beta(\mathbf{r}_{2B})$ of two one-electron wavefunctions (6). For large but finite R these functions have to be corrected to account for potentials of the Coulomb interactions of electrons between themselves and with the alien ions (7). Following [9, 10], one starts with the Schrödinger equation for the corrected functions $\psi_{\mu\nu}^{\alpha\beta}$

$$\hat{\mathcal{H}}_{\text{tot}}^{\alpha\alpha',\beta\beta'} \psi_{\mu\nu}^{\alpha'\beta'} = E_0 \psi_{\mu\nu}^{\alpha\beta}. \quad (8)$$

Let $\hat{\zeta}$ denote the direction from the donor A to the donor B, positioned at $\zeta = \mp R/2$ respectively. It turns out [9] that for the calculation of exchange interaction it is enough to know $\psi_{\mu\nu}$ on the median hyperplane $\zeta_1 = \zeta_2$ far from the centers. There one may seek for the functions $\psi_{\mu\nu}$ approximately in the form

$$\psi_{\mu\nu}^{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2) = \chi^{\alpha\alpha',\beta\beta'}(\mathbf{r}_1, \mathbf{r}_2) \varphi_\mu^{\alpha'}(\mathbf{r}_{1A}) \varphi_\nu^{\beta'}(\mathbf{r}_{2B}), \quad (9)$$

where $\chi^{\alpha\alpha',\beta\beta'}$ varies on the scale of the order of R .

Substituting (9) into (8) yields a differential equation on $\chi^{\alpha\alpha',\beta\beta'}$. The boundary condition is determined from the condition that when either $\mathbf{r}_{1A} \rightarrow 0$ or $\mathbf{r}_{2B} \rightarrow 0$

the function $\psi_{\mu\nu}^{\alpha\beta}$ should convert to $\varphi_\mu^\alpha(\mathbf{r}_{1A})\varphi_\nu^\beta(\mathbf{r}_{2B})$, i.e. $\chi^{\alpha\alpha',\beta\beta'} \rightarrow \delta^{\alpha\alpha'}\delta^{\beta\beta'}$. The principal terms in spin-orbital interaction are included into definition (6), and the equation for $\chi^{\alpha\alpha',\beta\beta'}$ turns out to coincide with the one in the absence of the spin-orbit [9]

$$\left(\partial_{\zeta_1} - \partial_{\zeta_2} - \frac{2}{R - 2\zeta_1} - \frac{2}{R + 2\zeta_2} + \frac{1}{R} + \frac{1}{r_{12}} \right) \chi = 0. \quad (10)$$

Here we used that $\partial\varphi \sim \varphi/a_B$ and hence one can neglect all but the first derivatives of the functions χ which varies on distances of order $R \gg a_B$.

Therefore $\chi^{\alpha\alpha',\beta\beta'} = \chi_0 \delta^{\alpha\alpha'} \delta^{\beta\beta'}$, where on the median 5-dimensional hyperplane $\zeta_1 = \zeta_2 \equiv \zeta$ [9]

$$\chi_0 = \frac{2R}{R + 2|\zeta|} \sqrt{\frac{2\rho_{12}}{R - 2|\zeta|}} e^{\frac{2|\zeta| - R}{2R}}. \quad (11)$$

Here $\rho_{12} = |\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2|$, and $\boldsymbol{\rho}_{1,2}$ are the transverse radii-vectors of the electrons.

The two-electron eigenfunction $\psi^{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2)$ of $\hat{\mathcal{H}}_{\text{tot}}$ adjusted for the exchange is then sought in the form

$$\psi^{\alpha\beta} = \sum_{\mu,\nu} c_{\mu\nu} \psi_{\mu\nu}^{\alpha\beta} + \sum_{\mu,\nu} c_{\nu\mu}^{(P)} \psi_{\nu\mu}^{\alpha\beta}, \quad (12)$$

where $c_{\mu\nu}$ and $c_{\mu\nu}^{(P)}$ are unknown coefficients. $\psi_{\nu\mu}^{\alpha\beta}$ stands for the interchanged states of the two electrons, when the first electron is now almost localized on atom B in the state ν and the second — on atom A in the state μ

$$\psi_{\nu\mu}^{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2) \equiv \psi_{\mu\nu}^{\beta\alpha}(\mathbf{r}_2, \mathbf{r}_1). \quad (13)$$

Because the total fermionic wavefunction should be antisymmetric in particle permutation $\psi^{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2) = -\psi^{\beta\alpha}(\mathbf{r}_2, \mathbf{r}_1)$, the coefficients $c_{\mu\nu}^{(P)} = -c_{\mu\nu}$.

We can now proceed to the derivation of the total exchange Hamiltonian. Multiplying the Schrödinger equation $E\psi = \hat{\mathcal{H}}\psi$ by $\psi_{\mu\nu}^*$ (for brevity we omit spinor structure), and the Schrödinger equation (8) $\hat{\mathcal{H}}^* \psi_{\mu\nu}^* = E_0 \psi_{\mu\nu}^*$ by ψ , subtracting term by term and integrating over the region $\zeta_1 < \zeta_2$, we find

$$(E - E_0)c_{\mu\nu} = \int_{\zeta_1 < \zeta_2} \left(\psi_{\mu\nu}^* \hat{\mathcal{H}}\psi - \psi \hat{\mathcal{H}}^* \psi_{\mu\nu}^* \right) d^6\mathbf{r}_{1,2}. \quad (14)$$

The kinetic part of $\hat{\mathcal{H}}$ can be readily reduced to a surface integral [9, 10]

$$(E - E_0)c_{\mu\nu} = \frac{1}{2} \int_{\zeta_1 = \zeta_2} (\psi \boldsymbol{\partial} \psi_{\mu\nu}^* - \psi_{\mu\nu}^* \boldsymbol{\partial} \psi) \mathbf{d}\mathbf{S} \quad (15)$$

over the median plane. Here $\mathbf{d}\mathbf{S} = (\hat{\zeta}_1, -\hat{\zeta}_2) d\zeta d^4\boldsymbol{\rho}_{1,2}$ is its surface element and $\boldsymbol{\partial} = (\partial_1, \partial_2)$ is the six-dimensional gradient.

We then substitute (12) into (15) neglecting derivatives of χ_0 compared to those of φ_μ :

$$\partial_{\zeta_{1,2}} \psi_{\mu\nu} \approx \mp \psi_{\mu\nu}, \quad \partial_{\zeta_{1,2}} \psi_{\nu\mu} \approx \pm \psi_{\nu\mu}. \quad (16)$$

After simple calculations Eq. (15) results in

$$\begin{aligned} (E - E_0)c_{\mu\nu} &= -2c_{\mu'\nu'}^{(P)} \int_{\zeta_1=\zeta_2} \psi_{\mu\nu}^* \psi_{\nu'\mu'} d\zeta d^4\mathbf{p}_{1,2} \quad (17) \\ &= J(R)(e^{-i\gamma\sigma/2})_{\mu\nu'}(e^{i\gamma\sigma/2})_{\nu'\mu'} c_{\mu'\nu'}^{(P)} \end{aligned}$$

with the angle

$$\gamma = m_e \hbar \mathbf{h}(\hat{\zeta}) R / a_B^2. \quad (18)$$

The exchange integral equals [9, 10]

$$\begin{aligned} J(R) &= -2 \int_{\zeta_1=\zeta_2} \chi_0^2 \varphi_0^*(r_{1A}) \varphi_0^*(r_{2B}) \varphi_0(r_{2A}) \varphi_0(r_{1B}) d\zeta d^4\mathbf{p}_{1,2} \\ &\approx -0.818 \times 2\text{Ry} (R/a_B)^{5/2} e^{-2R/a_B}. \quad (19) \end{aligned}$$

Using

$$c_{\mu\nu}^{(P)} = -c_{\mu\nu} \equiv -\frac{1}{2}(\sigma_{\mu\mu'}\sigma_{\nu\nu'} + \delta_{\mu\mu'}\delta_{\nu\nu'})c_{\nu'\mu'}. \quad (20)$$

we finally arrive at the equation for the coefficients $c_{\mu\nu}$, $(E - E_0)c = \hat{\mathcal{H}}_{\text{ex}}c$, with the Hamiltonian

$$\begin{aligned} \hat{\mathcal{H}}_{\text{ex}} &= -\frac{1}{2}J e^{-i\gamma(\sigma_1 - \sigma_2)/2} (\hat{\sigma}_1 \hat{\sigma}_2 + 1) \quad (21) \\ &\equiv -\frac{1}{2}J [\hat{\sigma}_1 \overset{\leftrightarrow}{R}(\gamma) \hat{\sigma}_2 + 1] \\ &\approx -\frac{1}{2}J [\hat{\sigma}_1 \hat{\sigma}_2 + 1 - \gamma (\hat{\sigma}_1 \times \hat{\sigma}_2)], \end{aligned}$$

where the Pauli matrices $\hat{\sigma}_1$ and $\hat{\sigma}_2$ are supposed to act in the space of the Kramers indices, and $\overset{\leftrightarrow}{R}(\gamma)$ is the three-dimensional rotation matrix on the angle γ around the axis $\hat{\gamma}$.

The rotation angle γ (18) gives the strength of the DM term relative to the isotropic exchange. It is perpendicular to axis $\hat{\zeta}$ since $\mathbf{h}(\hat{\zeta})\hat{\zeta} = 0$. Taking the square mean of $\mathbf{h}(\hat{\zeta})$ over the directions of $\hat{\zeta}$ we find that for GaAs

$$\bar{\gamma} = \langle \gamma^2 \rangle_{\hat{\zeta}}^{1/2} = \frac{2\alpha_{\text{so}}}{\sqrt{35E_g/Ry}} \frac{R}{a_B} \approx 0.00157 \frac{R}{a_B}. \quad (22)$$

This $\bar{\gamma}$ is shown in Fig. 1 against the R -dependence of $\bar{\gamma}$ from Ref. [7, Eq. (7)] for comparison. In the region of interest ($R \sim 3 \div 7a_B$) our result for $\bar{\gamma}$ is about one half that of Ref. [7, Eq. (7)].

We turn now to the question of how the antisymmetric exchange would manifest itself as a spin relaxation mechanism. It was suggested in Ref. [3, 7] to write down the corresponding relaxation time τ_{sa} in terms of a precession mechanism [12]:

$$\tau_{sa} = \frac{3}{2}\tau_c \bar{\gamma}^{-2}, \quad (23)$$

where now, unlike [12], $1/\tau_c$ is a rate of the flip-flop transition caused by isotropic exchange. Inconsistency of the physical picture of Ref. [3] in the use of the dynamical averaging in Eq. (23) was mentioned above. Another objectionable point is to write τ_c in the form [7]

$$\tau_c \approx \hbar/\xi J(R_c), \quad (24)$$

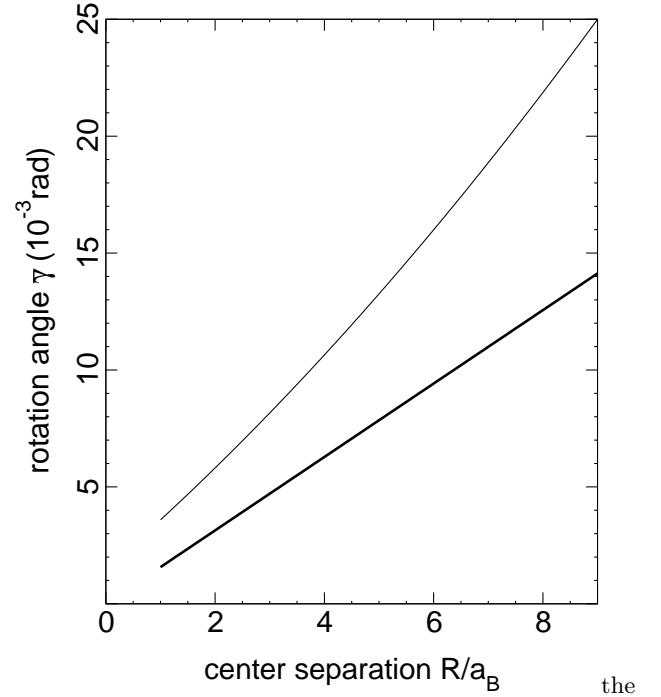


FIG. 1: Mean square rotation angles $\bar{\gamma}$ of the asymmetric interaction in GaAs as functions of the inter-center separations as calculated in the present paper (thick line) and according to the interpolation Eq. (7) in Ref. [7] (thin curve).

where $R_c \approx \beta n_D^{-1/3}$, ξ and β being the fitting parameters of the order of unity. The choice for the latter one, $\beta \approx 0.65$, interpolates the maxima in the Poisson distribution for the nearest and next nearest neighbors. Such a form is assumed in Ref. [7] to phenomenologically account for large average distances between donors. But since the maximum in the Poisson distribution is rather broad, it appears erroneous to merely substitute R_c for R in the exponential dependence $J(R)$.

At $n_D \approx 10^{16} \text{ cm}^{-3}$ the distance $R_c \approx 3.3a_B$, and from Fig. 1 for the angles $\bar{\gamma}$ [7, Eq. (7)] one obtains $\bar{\gamma}(R_c) \approx 0.009$, while the corrected value in Eq. (22) $\bar{\gamma} \approx 0.005$. The additional factor ≈ 4 in Eq. (23) immediately erodes the claimed quantitative agreement with experimental data in Ref. [1, 7].

With the applicability of (23) being questioned and the experimental value of τ_c being not determined in [7] unambiguously, we try to explore spin relaxation due to DM terms as a question of the EPR line shape for a dilute system of paramagnetic spins interacting via Hamiltonian (21). This appears a proper approach if all other sources of inhomogeneous line broadening are neglected and the limit is taken of the Larmor frequency $\omega_L = g\mu_B B \rightarrow 0$.

In Ref. [13] such a problem dealing with DM interaction has already been addressed for Mn-based II-VI-compound diluted magnetic semiconductors in the framework of *high temperature moment expansion* of the linear

response function

$$\chi''(\omega) = \frac{1}{2} \int_{-\infty}^{\infty} e^{-i\omega t} \langle [M_x(t), M_x(0)] \rangle dt. \quad (25)$$

Here square brackets denote quantum commutator of the magnetization operator $\mathbf{M} = g\mu_B \sum_n \mathbf{s}_n$ (summation runs over all donors). Angle brackets denote both thermodynamical average and averaging over disorder.

The spin relaxation time is then given by [13]

$$\tau_{sa} = \sqrt{2I_4/\pi I_2^3}, \quad (26)$$

where the frequency moments of the response function $I_n = \int_{-\infty}^{\infty} \omega^{n-1} \chi''(\omega) d\omega / \pi \chi(0)$ obtained in Ref. [13] as a high-temperature expansion, are

$$I_2 = -\text{Tr}([\hat{\mathcal{H}}_{\text{DM}}, M_x]^2) / \text{Tr}(M_x^2) + O(T^{-1}), \quad (27)$$

$$I_4 = \text{Tr}([\hat{\mathcal{H}}_{\text{ex}}, [\hat{\mathcal{H}}_{\text{DM}}, M_x]^2]) / \text{Tr}(M_x^2) + O(T^{-1}). \quad (28)$$

Calculating the commutators we find

$$I_2 = 4n_D \int J^2(r) [\gamma^2(\hat{\mathbf{r}}) - \gamma_x^2(\hat{\mathbf{r}})] d^3\mathbf{r} + O(T^{-1}), \quad (29)$$

$$I_4 \approx 32n_D \int J^4(r) [3\gamma^2(\hat{\mathbf{r}}) - \gamma_x^2(\hat{\mathbf{r}})] d^3\mathbf{r} + O(T^{-1}) \quad (30)$$

(For an order of magnitude estimate below we neglected in (30) the terms with double integration under the pretext that they are of the “second order” in n_D . Averaging over $\hat{\mathbf{r}}$ we use $\langle \gamma_i(\hat{\mathbf{r}}) \gamma_j(\hat{\mathbf{r}}) \rangle_{\hat{\mathbf{r}}} = \frac{1}{3} \bar{\gamma}^2 \delta_{ij}$)

The high-temperature expansion (29), (30), where one would integrate over all randomly distributed centers, applies to weakly interacting spins only. Eqs. (29), (30) need to be corrected to take into account that if two spins are so close that their exchange interaction $J(R) \gg T$, they would form a singlet state and drop out of the system’s thermodynamics. A proper calculation of the temperature effects in the EPR line due to clusterization should proceed in the framework of a scaling theory [14]. The problem is difficult and remains unsolved. We take the effects into account qualitatively introducing a cut-off in the integrals (29), (30) at short distances a_T found from the condition $J(a_T) = T$. (At $T = 2$ K this radius $a_T \approx 3.7a_B$, at $T = 4.2$ K $a_T \approx 3.1a_B$).

Calculation of (29), (30) yields the answer in terms of incomplete gamma functions $\Gamma(n, x)$:

$$\tau_{sa} = \frac{6}{\pi n_D a_B^3 0.818 \text{Ry} \bar{\gamma}^2} \sqrt{\frac{2\Gamma(13, 8a_T/a_B)}{\pi \Gamma^3(8, 4a_T/a_B)}}. \quad (31)$$

For GaAs at $n_D = 10^{16}$ cm⁻³ at $T = 2$ K we find $\tau_{sa} \approx 290$ ns, and at $T = 4.2$ K we find $\tau_{sa} \approx 170$ ns as compared to $\tau_s \approx 80$ ns found experimentally. (Using expression (23) with corrected value for γ from Fig. 1 would give the temperature-independent value of $\tau_{sa} \approx 330$ ns at $n_D = 10^{16}$ cm⁻³)

The method of taking a spin-orbital interaction into account, developed in the present paper to shallow donor centers in *bulk* zinc-blende semiconductors, may be easily generalized. In a low-dimensional environment, e.g., in quantum dots, spin-orbital interaction is described by a linear dependence on \mathbf{p} : $h_i(\mathbf{p}) = h_{ik} p_i$, where h_{ik} is a tensor. The spin structure of an exchange Hamiltonian (21) acting on the coefficients of expansion (12) again decouples from the coordinate dependence $J(R)$. Although explicit expression for $J(R)$, of course, now depends on the potential of the quantum dots, the form of the “rotated” exchange Hamiltonian (21) with the rotation angle $\gamma_k = m_e \hbar h_{ik} \hat{\zeta}_k R / a_B^2$ instead of (18) remains. For a specific choice of the linear in \mathbf{p} spin-orbital interaction this result may be obtained by an exact unitary transformation as has been recently shown in [16].

In the end, we comment on the role of the spin-orbital interaction of electrons with the electric field of alien ions. For a homogeneous field it has the form [15] $\alpha_E \mu_B g \hat{\mathbf{s}} \cdot (\hat{\mathbf{p}} \times \mathbf{E}) 2c/E_g$, the coefficient $\alpha_E \sim 1$ depends on the band structure. This term is proportional to $\sim \alpha_E (\text{Ry}^2/E_g) (m_e/m_0)$, and is much smaller than the term (4) considered above that is proportional to $\sim \alpha_{\text{so}} \text{Ry} (\text{Ry}/E_g)^{1/2}$.

To summarize, we derived the asymptotically correct form of the spin Hamiltonian for two hydrogen-like donors in the *n*-doped GaAs. We applied the EPR line shape formalism to analyze spin dephasing times τ_{sa} due to the antisymmetric DM exchange. The rough estimate for τ_{sa} (neglecting the detailed low-temperature spin clusterization which would further increase τ_{sa}) yields values exceeding those observed experimentally. Although the concentration range $n_D^{-1/3} \sim a_B$ is very difficult for theory, our results suggest that the anisotropic exchange between the localization centers is not a prevailing mechanism of the spin relaxation in this concentration range.

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